The Prediction of the Work of Friction Force on the Arbitrary Path

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Abstract: In this paper we have calculated the work of friction force on the arbitrary path. In our method didn't use from energy conservative conceptions any way. The distinction of this procedure is that at least do decrease measurement on the path once. Thus we can fore casting the amount of work of friction force without information about speed of mover.

Introduction

The conception of work in classical mechanics is familiar. It's defined as (Halliday et al., 1992),

$$W = \int \vec{F} . \vec{dl} \tag{1}$$

We use of conception of energy conservative on the arbitrary trajectory to calculation of wasted force work. Whenever the energy do not conservative then,

$$W_{f_k} = \Delta E \tag{2}$$

So that W_{f_k} is work of friction force and ΔE is difference of total energy of system that $E = U + K$ and U is potential energy and K is kinetic energy. Almost in the all of the basic and advanced classical mechanics relation (2) is impressive to obtain wasted force work. But there is an important point in relation (2) that to get W_{f_k} we have to know the magnitude of velocity of path. As an example assume that we projected a body on the trajectory to calculation of wasted force work, now obviously we should know the second speed of body at final point. But we get W_{f_k} straight forwardly and without aid of energy conception.

Method

First we define original definition as following

$$W_{f_k} = \int \vec{f_k} . \vec{dl} \tag{3}$$

We know on the curved paths normal force, N, is variable. So f_k is changeable too then it seems the solution of (3) is difficult. Based on the montioned we seek to proper approach to remove this dilemma.

To calculation of f_k we have two general states:

1) $y = f_{(x)}, \frac{d^2y}{dx^2} > 0$

Pay attention to following figure

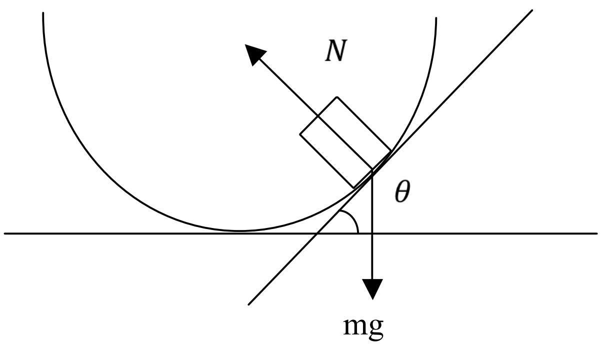

Fig.1. The illustration of one of the possible conditions

$$N = mg\cos\theta + \frac{mv^2}{r} \tag{4}$$

Where r is the radious of curvature of path.

2) $y = f_{(x)}$, $\frac{d^2y}{dx^2} < 0$

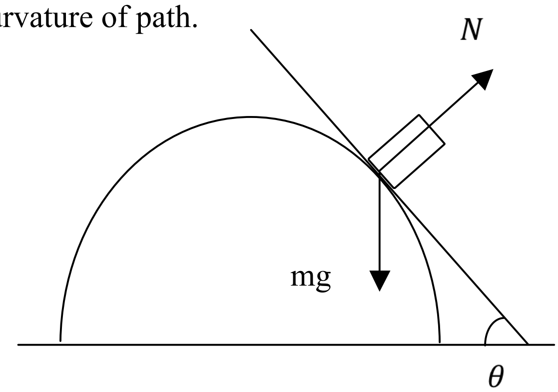

Fig.2. Shape of another possible condition.

$$N = mg\cos\theta - \frac{mv^2}{r} \tag{5}$$

Now f_k is equal to

$$f_k = \mu_k N = \mu_k (mg\cos\theta \pm m\frac{v^2}{r}) \tag{6}$$

Clearly in above equation $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$
Note that the motion of body on the trajectory such as figure (2) we have two other states

$$\begin{cases} \theta \ge 0 & \qquad f'_{(x)} \ge 0 \\ \theta < 0 & \qquad f'_{(x)} < 0 \end{cases} \tag{7}$$

And with due attention to

$$f'_{(x)} = \tan\theta \quad , \quad \theta = \tan^{-1}(f'_{(x)}) \quad , \quad f'_{(x)} = \frac{dy}{dx} \tag{8}$$

To rewrite relation (6) we use some mathematical points. By the (7) we have,

$$\cos\theta = \pm\frac{1}{\sqrt{1+\tan^2\theta}} \tag{9}$$

By preposition to condition $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ realation (8) will be choosen with positive sign.

$$\cos\theta = \frac{1}{\sqrt{1+(f'_{(x)})^2}} \quad , \quad f'_{(x)} = \frac{dy}{dx} \tag{10}$$

And we know for every arbitrary path with equation of $y = f_{(x)}$, we have,

$$l = \int_{l_1}^{l_2} \sqrt{1+(\frac{dy}{dx})^2}\, dx \tag{11}$$

By the derivative of (11) than (x) and with (10) we get

$$\frac{dl}{dx} = \sqrt{1+f'^{\,2}_{(x)}} = \frac{1}{\cos\theta} \tag{12}$$

So

$$\sin\theta = \pm\frac{\left|\frac{dy}{dx}\right|}{\frac{dl}{dx}} \tag{13}$$

If $\frac{dy}{dx} > 0$ then $\sin\theta > 0$ and If $\frac{dy}{dx} < 0$ then $\sin\theta < 0$

Now by the identify sign of (13) we can write,

$$\sin\theta = f'_{(x)} \times \cos\theta = \frac{\frac{dy}{dx}}{\frac{dl}{dx}} \tag{14}$$

Also, again (Thomas, 1959) the radious curvature r related equation $y = f_{(x)}$ obtain

$$r = \frac{(1+(\frac{dy}{dx})^2)^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} \tag{15}$$

By the equation (12) we get

$$r = \frac{(\frac{dl}{dx})^3}{\left|\frac{d^2y}{dx^2}\right|} \tag{16}$$

So equation (6) will be as following

$$f_k = \mu_k mg\cos\theta \pm \mu_k \frac{mV^2\left|\frac{d^2y}{dx^2}\right|}{(\frac{dl}{dx})^3} \tag{17}$$

If we conider V as instant velocity along motion path that is $V = \frac{dl}{dx}$ then (17) wil be as following

$$f_k = \mu_k(mg\cos\theta \pm m(\frac{dl}{dx})^2 \times \frac{\left|\frac{d^2y}{dx^2}\right|}{(\frac{dl}{dx})^3}) \tag{18}$$

Note that the path of motion is fully arbitrary so sign of $\frac{d^2y}{dx^2}$ can change again and again. Now we have two state in general. If $\frac{d^2y}{dx^2} > 0$ then should be choose positive sign in (18) and vice versa. Therefore relation (18) be choose by positive sign and without the magnitude. After some calculation we get,

$$f_k = \mu_k mg\cos\theta + \mu_k m\left(\frac{d^2y}{dt^2}\right)(\frac{dx}{dl}) \tag{19}$$

Where $m\left(\frac{d^2y}{dt^2}\right)$ is compnent of netforces along of y axis. Since sign of derivative of function can change then we have two following states

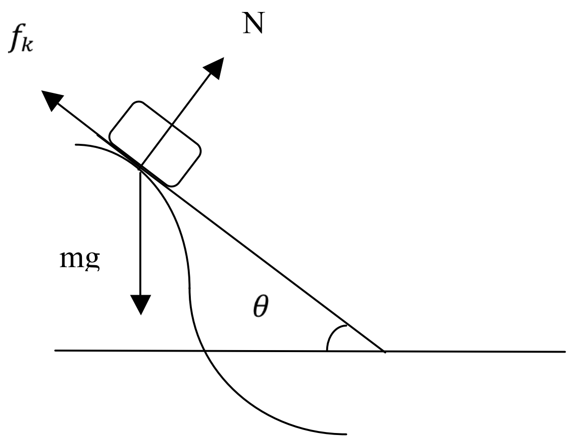

Fig. 3 The first state (a), θ is angle that the tangent line on the curve make with horizontal axis.

For state of (a), according above figure, we get

$$\frac{dy}{dx} < 0 \; \rightarrow m\frac{d^2y}{dt^2} = mg - N\cos\theta - |f_k \sin\theta|$$

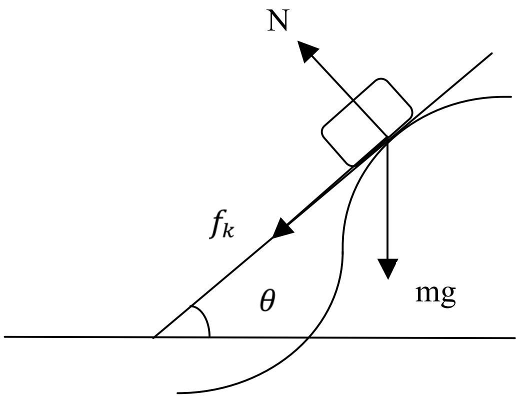

Fig. 4 The second state (b), θ is angle that the tangent line on the curve make with horizontal axis.

For state of (b), based on the figure (4), we get

$$\frac{dy}{dx} > 0 \; \rightarrow m\frac{d^2y}{dt^2} = mg - N\cos\theta + |f_k \sin\theta|$$

in (a) condition we have
$f'_{(x)} > 0 \rightarrow$
$\tan^{-1}(f'_{(x)}) < 0 \rightarrow \theta < 0 \rightarrow \sin\theta < 0 \rightarrow$ we can write $(\,f_k \sin\theta)$ instead of $(-|f_k \sin\theta|$
$)$
and in (b) condition we have
$f'_{(x)} < 0 \rightarrow$
$\tan^{-1}(f'_{(x)}) > 0 \rightarrow \theta > 0 \rightarrow \sin\theta > 0 \rightarrow$ we can write $(\,f_k \sin\theta)$ instead of $(+|f_k \sin\theta|$
$)$

so we have

$$m\frac{V^2}{r} = m\frac{d^2y}{dt^2} \times \frac{dx}{dl} = m\frac{d^2y}{dt^2}\cos\theta = \frac{mg - mg\cos^2\theta + f_k \sin\theta}{1 + \cos^2\theta}\cos\theta \qquad (20)$$

Then by putting (20) in (19) we get

$$f_k = \frac{\mu_k mg\left(\frac{2}{1+\cos^2\theta}\right)\cos\theta}{1 - \mu_k\frac{\sin\theta\cos\theta}{1+\cos^2\theta}} \qquad (21)$$

Now according to $\cos\theta = \frac{dx}{dl}$ and putting (21) in relation (3) we obtain final equation

$$W_{f_k} = \int_{l_1}^{l_2} \frac{(2\mu_k mg)\frac{dx}{dl}}{1+cos^2\theta-\mu_k \sin\theta\cos\theta} dl \qquad (22)$$

Finally by relations (10) and (14) we have work of friction force on the arbitrary path.

$$W_{f_k} = 2\mu_k mg \int_{x_1}^{x_2} \frac{1+(f'_{(x)})^2}{2+(f'_{(x)})^2-\mu_k f'_{(x)}} dx \qquad (23)$$

Conclusion

We have tried to make a formula to calculate work of friction force directly. Relation (23) indicates that for this purpose we don't need to use energy relations. In addition to this, in our method the measurement on the system decreases. Because for the calculation of work of friction force no need to obtain final speed of body on the path. Instead it's sufficient to know the equation of path, mass of body and coefficient of friction. In fact, we can anticipate that work of friction force without knowing the second speed atfinal point.

References

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