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Teaching the Falling Ball Problem with Dimensional Analysis

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Abstract
Dimensional analysis is often a subject reserved for students of fluid mechanics. However, the principles of scaling and dimensional analysis are applicable to various physical problems, many of which can be introduced early on in a university physics curriculum. Here, we revisit one of the best-known examples from a first course in classic mechanics, namely the falling ball problem: a ball is thrown with an initial velocity and height while experiencing gravity and viscous drag. We treat two representative cases of drag forces, one linear and one quadratic in velocity. We demonstrate that the ball's motion is governed by two dimensionless parameters: (i) a Froude number (Fr) comparing the ball's initial kinetic to potential energy and (ii) a drag coefficient (C_D) comparing the initial drag force to the ball's own weight. By investigating extreme, yet simple hypothetical cases for Fr and C_D, we demonstrate how students can grasp the role of the parameters relating the ball's initial conditions in governing several physical behaviors displayed by the system. Advocating early on exposure to dimensional analysis is undoubtedly beneficial in building physical intuition, but also it illustrates how physical systems characterized by many variables may be assimilated by reducing their inherent complexity.
Keywords: Dimensional analysis; scaling; dimensionless parameters; classic mechanics; Newton's laws.

Introduction

In traditional curriculums of university physics, all too rarely are students exposed to the concept of dimensional analysis and dimensionless parameters. Even engineering students must typically wait until their first exposure to a fluid mechanics course (Fox et al., 2008; Panton, 1995; Smits, 2000) to discover the existence of the Buckingham-Pi theorem and dimensionless parameters such as the ubiquitous Reynolds number. A full-scale treatment of dimensional analysis is often reserved to graduate-level textbooks (Bridgman, 1946; Gibbings, 2011; Szirtes, 2007). However, dimensional analysis is usually a simple, yet powerful tool to help gain insight into a wide variety of physical phenomena (Buchanan, 2010), ranging from printing and painting (Herczynski et al., 2011) to nature's designs of organisms (Vogel, 1998). Indeed, a first course in college physics (Giancoli,

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2008) is rife with examples where dimensional analysis may be introduced and resulting dimensionless parameters deduced. Such an exercise can provide students some added insight into the physics governing a system, with little or no additional complexity in the algebra needed. In fact, nearly a century ago Lord Rayleigh (Rayleigh, 1915) outlined what he termed the principles of similitude through "*a few examples, chosen almost at random from various fields, (that) may help to direct the attention of workers and teachers to the great importance of the principle*."

In the present article, our pedagogical approach lies hence in revisiting one of the best-known first-year undergraduate examples of classical mechanics: namely, the falling ball problem. With this simple example, we hope to convey to the instructor, and ultimately his/her students, some insight into the power of dimensional analysis. Indeed, students will realize that the simple falling ball problem may be better understood by determining directly from the force balance which non-dimensional groups govern the system's dynamics. From an experimentalist's point of view, this strategy is particularly valuable in an effort to decrease the inherent complexity of a multi-variable problem leading to prohibitively large numbers of required experiments (Bolster et al., 2011; Owen and Ryu, 2005). By knowing beforehand which dimensionless groups characterize a physical system, students can easily determine the parameters governing measurable outcomes, even prior to solving explicitly the characteristic differential equations. In particular, we will illustrate here one specific aspect that highlights the usefulness of dimensional analysis: how initial conditions (i.e. initial height and velocity) influence the ball's resulting trajectory. Altogether, our approach is intended to lead a student through the application of dimensional analysis, while discussing the problem physically and graphing several of the solutions illustrating the impact of each dimensionless number, respectively.

A schematic illustrating the falling ball setup is shown in figure 1. Namely, a ball of mass m is positioned at time t=0 at a height $y=h_o$ with a downward initial velocity of magnitude $|v(t=0)|=|v_o|$, and experiences the influence of the gravitational field $\vec{g}$ acting in the negative y-direction. Additionally, the ball is subject to viscous resistance (i.e. drag) due to the surrounding fluid. Here, the fluid (e.g. air) is assumed to have a density much smaller than that of the object such that buoyancy effects may be entirely neglected. The drag force experienced by the ball due to air may take different algebraic forms and we discuss briefly two classic cases: (i) drag proportional to velocity (i.e. *linear drag*) and (ii) drag proportional to velocity squared (i.e. *quadratic drag*).

Grasping the concept of scaling

In an effort to introduce to students the power of dimensionless analysis, let us first consider for a moment the simplest case where the ball falls in the absence of friction. One classic question from a first-year physics course pertains to estimating the time (T_i) for the ball to impact the ground, if the object is released from rest at height h_o. In such an idealized situation, the system is characterized by the variables h_o, g and m (see Table 1). Hence, we are seeking the impact time T_i that must be constructed from the identified variables and their corresponding fundamental dimensions (i.e. mass [M], time [T] and length [L]). Since h_o has dimension [L] (e.g. with S.I. units of [m]), g has dimensions [L/T^2] (e.g. with S.I. units of [m/s^2]) and m has dimension [M] (e.g. with S.I. units of [kg]), we observe that since the only variable with dimension [M] is the mass, this variable cannot enter our final formula. Hence, to have an answer that has dimension [T] we must have T_i = (constant) · $(h_o/g)^{1/2}$ ($\equiv$ ([L]/[L/T^2])$^{1/2}$ = [T]). Note that to determine or estimate the value of the constant either requires conducting an experiment or a detailed calculation; yet, in typical circumstances we expect the magnitude of this constant to be approximately 1 (i.e. O (1)).

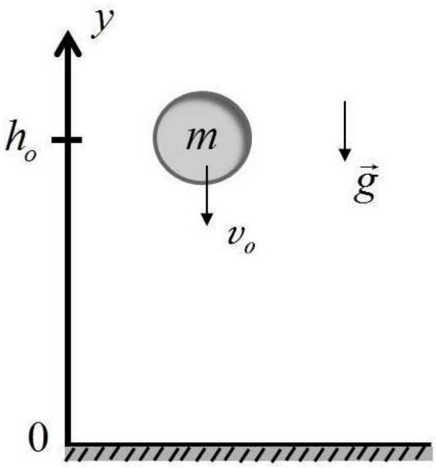

***Figure 1**: A ball of mass m falling under the influence of gravity ($\vec{g}$). The ball is thrown with a downward velocity of magnitude $|v_o|$ from an initial height h_o and is subject to friction (k) imposed by the surrounding fluid (e.g. air) of viscosity μ.*

In other words, we find that the time for the ball to touch the ground is independent of its mass. Moreover, increasing the initial height by a factor of 4 increases the time to contact by a factor of 2. This simple exercise illustrates the concept of fundamental dimensions and scaling where we have obtained a quantitative answer without even considering a differential equation! To proceed further in our analysis and include additional variables (e.g. initial speed, viscous drag), we now turn to applying the concepts of scaling and non-dimensionalization to ordinary differential equations (ODEs) introduced in a first-year undergraduate physics class.

Case I: Linear drag

Here, we will first assume that the drag force experienced by the ball is of the form $F_v = kv$, where the velocity is given by $v=dy/dt$ and the friction coefficient (k) resulting from the fluid's viscosity must have dimensions [M/T] (with S.I. units [kg/s]) such that F_v is a force (e.g. Newtons). Note that the linear drag form is best known to describe drag experienced by small objects (e.g. particles) in a viscous environment and is often referred to as *Stokes' law* (Batchelor, 2000) when considering spherical particles (i.e. $k = 3\pi\mu d$, where μ is the fluid viscosity and d the particle diameter). In the language of dimensional analysis, Stokes' law is typically accurate for small values of the dimensionless Reynolds number (i.e. $Re = \rho u d/\mu << 1$, where ρ is the fluid density and u is the characteristic speed of the particle), which compares the magnitude of inertial to viscous forces of the flow surrounding the object.

***Table 1**: List of variables, their fundamental dimensions and corresponding S.I. units.*

Variable	Definition	Dimensions	Units
y	position	[L]	[m]
t	time	[T]	[s]
m	mass	[M]	[kg]
g	gravitational acceleration	[L/T^2]	[m/s^2]
h_o	initial height	[L]	[m]
v_o	initial velocity	[L/T]	[m/s]
k	friction coefficient (linear)	[M/T]	[kg/s]
	or		

	friction coefficient (quadratic)	[M/L]	[kg/m]

As the ball is thrown with an initial downward velocity, the drag force acts in the opposite direction to the ball's motion: in the chosen coordinate system F_v must act in the positive y-direction, with v_o<0. We begin by writing and rearranging the force balance for the ball falling along the vertical y-axis (see schematic of figure 1):

$$m\frac{d^2y}{dt^2} = F_g + F_v, \qquad (1)$$

$$m\frac{d^2y}{dt^2} = -mg - kv, \qquad (2)$$

$$\frac{d^2y}{dt^2} + \frac{k}{m}\frac{dy}{dt} = -g, \qquad (3)$$

where v<0 along the y-axis. The above equation is a second-order, linear ODE. While students may often be able to solve the homogeneous and particular solutions of equation (3), and then use the appropriate initial conditions to solve for the resulting integration constants, these mathematical steps are not necessarily synonymous with ensuring that students grasp the physical role of the system's parameters (and initial conditions) in governing the ODE and the resulting dynamics of motion. This is precisely where dimensional analysis is anticipated to help.

To render equation (3) dimensionless requires first non-dimensionalizing the dependent and independent variables of the system, namely y and t. That is, we must construct scaling relationships for position and time by introducing the ratios

$$Y = \frac{y}{l_c}, \qquad \tau = \frac{t}{t_c}, \qquad (4)$$

where Y and τ are the dimensionless position and time, respectively. Here, l_c and t_c are often referred to as the characteristic *length scale* and *time scale*, and are typically constructed using the identified parameters of the system (Table 1). Given the dimensions of the available parameters, there is frequently more than one correct way to define Y and τ. Since we are specifically interested in using dimensional analysis to characterize the role of the initial conditions, we introduce[2] here $l_c = h_o$ and $t_c = h_o/|v_o|$. As a result, the dimensionless position is bounded between Y=1 when the ball begins at height h_o, and Y=0 when the ball hits the ground. The expression for t_c represents the time needed for the ball to fall a distance h_o at constant speed $|v_o|$. Inserting the dimensionless variables Y and τ into equation (3) yields:

$$\frac{d^2(h_oY)}{d(h_o\tau/|v_o|)^2} + \frac{k}{m}\frac{d(h_oY)}{d(h_o\tau/|v_o|)} = -g,$$

[2] Alternatively, one could choose for example t_c = (terminal velocity)/g = m/k and $l_c = gt_c^2$ to omit use of initial conditions during the scaling step. However, this approach yields a characteristic dimensionless ODE that does not feature any dimensionless parameters (i.e. $d^2Y/d\tau^2 + dY/d\tau = -1$), and thus renders it difficult for students to sample the importance of initial conditions, and evaluate extreme scenarios, without obtaining explicitly the ODE's solution and solving for the integration constants.

$$\frac{v_o^2}{gh_o}\frac{d^2Y}{d\tau^2}+\frac{k|v_o|}{mg}\frac{dY}{d\tau}=-1. \qquad (5)$$

All the individual terms of equation (5) are now non-dimensional and the resulting dimensionless ODE may be rewritten as

$$Fr\frac{d^2Y}{d\tau^2}+C_D\frac{dY}{d\tau}=-1, \qquad (6)$$

where we have introduced

$$Fr=\frac{v_o^2}{gh_o}=\frac{mv_o^2}{mgh_o}\quad\frac{\text{initial kinetic energy}}{\text{initial potential energy}}, \qquad (7)$$

$$C_D=\frac{k|v_o|}{mg}\quad\frac{\text{initial drag force}}{\text{ball's weight}}. \qquad (8)$$

The dimensionless parameter Fr is known in fluid mechanics as the *Froude number*, named after the English engineer William Froude (1810-1879) who was the first to formulate reliable laws for the resistance that water offers to ships and for predicting their stability. In particular, the numerator of Fr is proportional to the initial kinetic energy (mv_o^2) while the denominator is proportional to the initial potential energy (mgh_o). In contrast, the dimensionless drag coefficient (C_D), as defined here[3], represents the ratio of the initial magnitude of the drag force at time t=0 ($|v_o|k$ with dimensions [L/T] x [M/T] = [ML/T^2]) to the ball's weight (mg). These two forces appear in the original (dimensional) force balance.

As noted earlier, the scaling choices introduced in equation (4) serve the specific purpose of illustrating the influence of the ball's initial conditions (i.e. h_o and v_o). It is important to emphasize, however, that for the simple case of a ball starting from rest (v_o =0), the dimensionless parameters yield $Fr = C_D = 0$ and equation (6) breaks down mathematically. That is, equation (6) is only valid for cases where strictly speaking $v_o \neq 0$ (and equally m, g, and h_o are all non-zero). Indeed, the most general non-dimensional ODE characterizing the ball's dynamics is $d^2Y/d\tau^2 + dY/d\tau = -1$ for the specific choices[2] of $t_c = m/k$ and $l_c = gm^2/k^2$. However, in the absence of any dimensionless parameter arising from the non-dimensionalization step, this latter ODE would require solving for the explicit (analytical) solution in order for students to grasp some of the underlying physics of the problem. With this limitation in mind, we restrict the discussion below instead to cases where $v_o \neq 0$.

One formidable outcome of the non-dimensionalization exercise leading to equation (6) lies in the dramatic reduction of the total number of variables the system is described by. Initially, we began with a dimensional system represented by 7 parameters (see Table 1). In non-dimensional form, the falling ball problem is now described by 4 dimensionless parameters (i.e. Y, τ, Fr, and C_D). In particular, the coefficients Fr and C_D affect the relative importance of the dimensionless acceleration (i.e. $d^2Y/d\tau^2$) and the velocity (i.e. $dY/d\tau$), respectively. That is, the relative importance of the time-dependent terms may be assessed by considering the ratio

[3] In general, the notation C_D refers in classic fluid dynamics (Smits, 2000) to the drag coefficient defined as $C_D = F_v/(\rho v^2 A/2)$, where A is a reference area (e.g. cross-sectional area of the ball). Here, we have chosen to keep the same symbol (C_D) only to emphasize to students that we are introducing a dimensionless ratio that also scales the viscous drag force. In the falling ball problem, however, the drag force is scaled by the ball's weight (mg) rather than by the force produced by the dynamic pressure ($\rho v^2/2$) times the area (A), as commonly used in aerodynamics.

$$\frac{Fr(d^2Y/d\tau^2)}{C_D(dY/d\,\tau)}. \qquad (9)$$

Equation (9) highlights that alone the dimensionless coefficients Fr and C_D cannot determine which terms to neglect in equation (6). This property is best illustrated by considering the ball's acceleration. Namely, the term $d^2Y/d\tau^2$ will be important even though Fr may be small[4]. This singularity results from explicitly choosing the non-dimensionalization parameters to feature the initial conditions of the problem (i.e. h_o and v_o). As a result, there will always be a (very) short time where no matter how small values of Fr are taken, the second derivative term in equation (6) will be important[4] in deviating the ball's trajectory from the solutions captured by the simpler first-order ODEs (described below). However, within the limited scope of our discussion and to help students gain in a first step physical insight into the respective roles of Fr and C_D, we will address briefly two simple, yet illustrative cases restricted to “extreme”' scenarios only.

Note that the pedagogical approach adopted here follows in spirit traditional introductions to dimensionless analysis in fluid dynamics at the undergraduate level (Panton, 1995; Smits, 2000). Namely, two well-known simplified equations arise by considering solely the magnitude of the Reynolds number in the dimensionless Navier-Stokes (momentum) equations of an incompressible flow (ρ=constant), without considering changes in the time-dependent flow terms (i.e. unsteady and/or convective acceleration): (i) the inviscid (μ=0) Euler equations at high Reynolds numbers (Re >> 1) and (ii) the viscous Stokes' equations for creeping flow (Re << 1). In analogy to such fluid dynamic treatments, we limit ourselves for the falling ball problem to changes in Fr and C_D only, while changes in the time-dependent terms (i.e. $d^2Y/d\tau^2$ and $dY/d\tau$) are not considered. As a result, two simple hypothetical cases follow:

(i) In the first scenario, we consider the magnitude of C_D to be negligible corresponding to situations where the initial drag force is much smaller than the ball's weight (i.e. $|v_o|k << mg$). For such cases, equation (6) reduces to:

$$Fr\frac{d^2Y}{d\tau^2} = -1. \qquad (10)$$

Students will quickly realize that the ball's position is then captured by a characteristic quadratic solution, since the ball must continuously accelerate under the influence of its own weight. Noticeably, the ball's downward speed is inversely proportional to Fr; the smaller Fr, the faster the ball falls and the less time needed to touch the ground (Y=0). This may be understood from figure 2, where profiles of Y(τ) are shown for a range of values of Fr, and the ball's speed corresponds to the slope of Y(τ).

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[4] In fact, the smaller Fr the larger $d^2Y/d\tau^2$ at times near $\tau = 0$. This is best understood from solving explicitly equation (6) given the appropriate boundary conditions: $Y(\tau = 0) = 1$ and $dY(\tau = 0)/d\tau = -1$. Solving analytically the second-order linear ODE for the homogeneous and particular solutions yields $Y(\tau) = C_D/Fr \cdot (1-1/C_D) \cdot (exp(-C_D/Fr \cdot \tau) -1/C_D) + (1- \tau /C_D)$. In turn, at $\tau = 0$ the second derivative is given by $d^2Y(\tau = 0)/d\tau^2 = C_D/Fr \cdot (1-1/C_D)$. As $Fr \rightarrow 0$, the term $d^2Y(\tau = 0)/d\tau$ will become increasingly large for a fixed value of C_D.

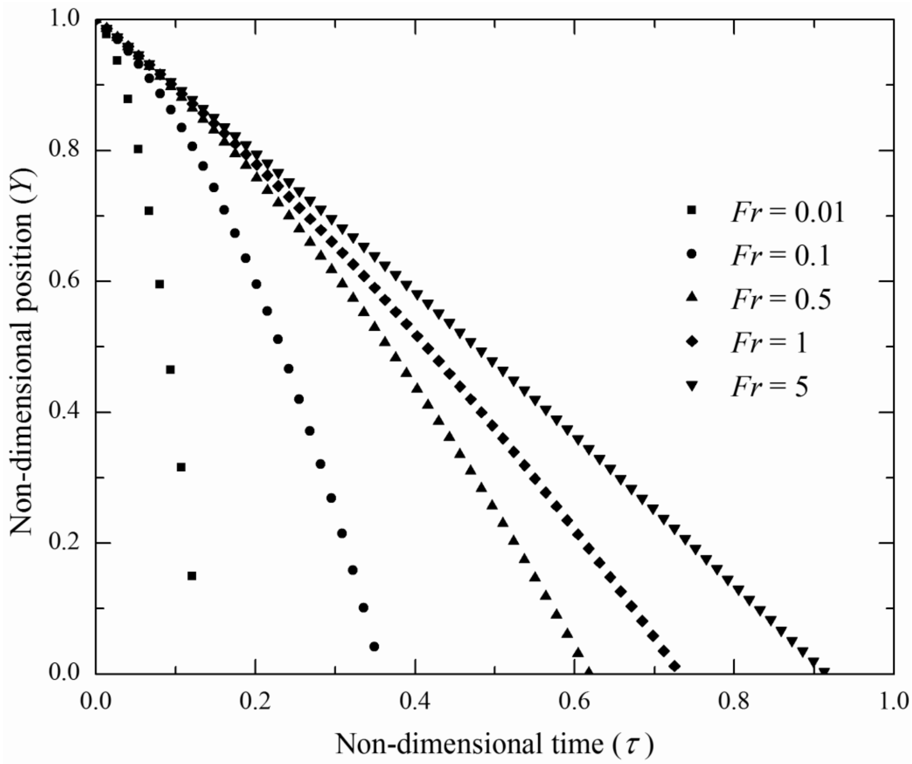

Figure 2*: Non-dimensional trajectories of the falling ball for scenarios when the dimensionless drag coefficient is negligible (C_D << 1 for linear drag, and c_d << 1 for quadratic drag). The dimensionless Froude number (Fr) controls the ball's parabolic trajectory and its resulting speed since (linear or quadratic) drag is negligible.*

(ii)In the second scenario, we consider instead the case where Fr is negligible. For instance, one can think of the extreme situation where the magnitude of h_o is dramatically increased compared to fixed values of $|v_o|$ and g. As a result, equation (6) reduces to:

$$C_D \frac{dY}{d\tau} = -1. \qquad (11)$$

Mathematically, the equation above yields a constant speed, namely $dY/d\tau = V = -1/C_D = -mg/|v_o|k$. This dimensionless speed may be best understood by realizing that it corresponds to the ratio of the ball's terminal velocity to initial velocity (i.e. $-|v_t|/|v_o|=-(mg/k)/|v_o|$), where $|v_t|$ is determined by solving equation (3) under steady-state conditions (i.e. when acceleration is zero and viscous forces balance gravitational forces). Students will immediately grasp that the ball's position is then captured by a characteristic linear profile upon integrating equation (11). Examples of such profiles are shown in figure 3 for a range of values of C_D. Physically, one can think of the following scenario: if the ball's initial position is very high, the relative distance needed for the ball to reach its steady-state velocity (V) is small compared to the total distance (h_o) the ball falls.

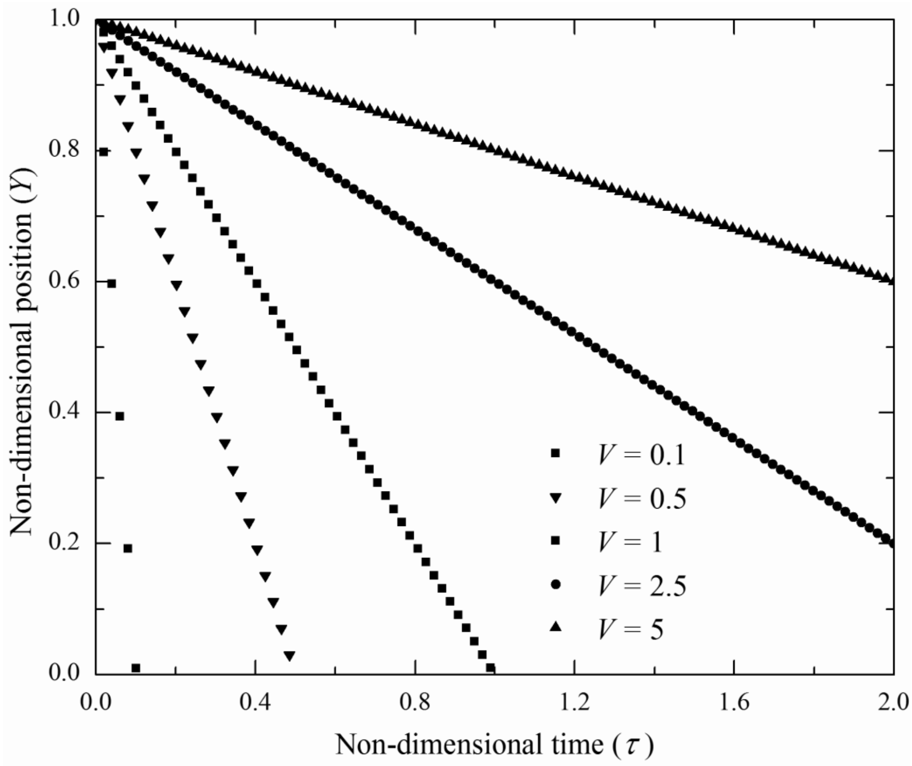

***Figure 3**: Non-dimensional trajectories of the falling ball for scenarios when Fr << 1. The ball reaches quasi-instantly a constant steady-state speed $V = -1/C_D$. Both linear and quadratic drag cases illustrate qualitatively identical profiles since V is inversely proportional to C_D and c_d, respectively.*

Case II: quadratic drag

Let us now investigate briefly the effects of the drag force when it is instead proportional to velocity squared, and thus takes the form $F_v = kv^2$ (note that k now has dimensions [M/L] and S.I. units [kg/m]). This formulation of drag (i.e. the so-called *drag equation*) describes objects typically moving through fluids at relatively large velocities (Batchelor, 2000), which corresponds to large values of the Reynolds number (*Re* > 2000). Within the frame of our discussion, the main question lies here in answering how the quadratic drag formulation will affect the resulting dimensionless parameters governing the ball's motion. We begin by writing explicitly the modified force balance:

$$m\frac{d^2y}{dt^2} = k\left(\frac{dy}{dt}\right)^2 - mg,$$

$$\frac{d^2y}{dt^2} - \frac{k}{m}\left(\frac{dy}{dt}\right)^2 = -g. \qquad (12)$$

Since the expression for drag is now quadratic in velocity, the drag force (F_v) has a positive sign, opposing downward motion due to gravity. Rendering the above equation non-dimensional, we introduce the same scaling relationships for the dimensionless position (Y) and time (τ) as chosen for the linear drag scenario. Namely, the characteristic length ($l_c=h_o$) and time ($t_c = h_o/|v_o|$) scales remain unchanged since the initial conditions are identical to those for linear drag. Substituting for Y and τ in equation (12) yields:

$$\frac{v_o^2}{gh_o}\frac{d^2Y}{d\tau^2} - \frac{v_o^2 k}{mg}\left(\frac{dY}{d\tau}\right)^2 = -1,$$

$$Fr\frac{d^2Y}{d\tau^2} - c_d\left(\frac{dY}{d\tau}\right)^2 = -1, \qquad (13)$$

where the drag coefficient now takes the form

$$c_d = \frac{v_o^2 k}{mg}. \qquad (14)$$

Here, we have introduced for clarity the symbol c_d to differentiate between the expressions in equations (8) and (14). In analogy to our earlier results (equation 9), the coefficients Fr and c_d influence the relative importance of the acceleration and the velocity in equation (13), respectively. Notice that the Froude number is unchanged from the linear drag form since Fr is unrelated to the expression for viscous drag. However, the dimensionless drag coefficient (c_d) is slightly modified since the ball's initial drag force is now $v_o^2 k$, following the modified dimensions of k for quadratic drag (see Table 1). In turn, the ball's terminal velocity is now expressed as $|v_t| = (mg/k)^{1/2}$, upon solving equation (13) under steady-state conditions.

Following the same strategy highlighted for the linear drag case, we can obtain some physical intuition into the modified dynamics of the ball under quadratic drag by limiting ourselves again to two extreme, yet simple situations:

(i) For scenarios where c_d is negligible (i.e. $v_o^2 k << mg$), equation (13) falls back to equation (10) introduced for linear drag. In this hypothetical case when Fr has non-zero values (i.e. $v_o \neq 0$), $c_d = 0$ will occur when viscous drag is negligible (i.e. $k \rightarrow 0$). As a result, the ball simply accelerates under its own weight with the well-known parabolic solution discussed earlier (see figure 2).

(ii) If instead Fr is negligible as discussed for linear drag (e.g. the extreme case where h_o is dramatically increased relative to $|v_o|$ and g), equation (13) reduces to:

$$c_d\left(\frac{dY}{d\tau}\right)^2 = 1. \qquad (15)$$

This characteristic equation is straightforward to grasp and students will recognize the linear solution $Y(\tau) = 1 - c_d^{-1/2}\tau$, recalling that $dY/d\tau < 0$. In such case, the ball's acceleration is negligible and the ball reaches quasi-instantly its dimensionless steady-state velocity (i.e. $|v_t|/|v_o| = (mg/k)^{1/2}/|v_o|$). Hence, the ball's trajectory will follow qualitatively the same profiles illustrated for linear drag (see figure 3).

Conclusions

The analysis described above serves a dual purpose: on the one hand, addressing the classic falling ball problem through dimensional analysis is helpful for students to assimilate physical concepts. Conversely, the concept of dimensional analysis itself may be introduced in a straightforward fashion through the falling ball problem, with little or no added complexity in the mathematics handled. The rational for introducing dimensional analysis at an early stage in the curriculum of physics and engineering students has some important advantages. To begin, dimensionless parameters provide insight into the physical mechanisms governing a dynamic system without explicit knowledge of the solution to the characteristic ODEs. In addition, faced with the same mathematical tools needed to solve the dimensional case (i.e. solving ODEs), the dimensionless treatment of the problem captures at a glance a number of extreme cases highlighting which variables of a system are critical to consider. While these simple scenarios are often unable to capture the entire range of dynamics the system can display, they are nevertheless paramount in helping students build physical intuition and inciting them to carry out in a first step "back of the envelope" calculations.

Overall, our discussion advocates more use of dimensional analysis in general introductory university/college physics classes. It is often a pity to have to wait until a second- or third-year specialized undergraduate fluid mechanics course to discover the existence of dimensionless parameters and the advantages of treating physical problems in dimensionless form. In particular, experimentalists, whether they are engineers or physicists, can instantly appreciate the power of dimensional analysis in illustrating the minimal necessary experiments needed to capture the dynamics of a system.

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