Determination of the Declination of the Sun on a Given Day

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Abstract
In this article we discuss a novel and low cost experiment to determine the angle of declination of the sun on a chosen day. The declination of the sun varies from + 23.5 degrees to - 23.5 degrees in the course of one year. If the zenith angle of the sun is measured as a function of its hour angle on a particular day, it is possible to calculate the angle of declination by using of spherical trigonometric relationship between them. The zenith angle is determined by using the shadow length of a vertically placed rod. The hour angle is determined from the position of shadow as a function of time. The value of solar declination may be compared with that available in almanacs.
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Introduction

The declination angle of the sun, denoted by δ , is equal to the angle between the solar position and the celestial equator measured along the great circle containing the sun and the celestial poles. It varies from +23.5 degrees to $-$ 23.5 degrees in the course of one year. It is equal to zero on the equinoxes. On a given day, it may be taken to be a constant. The point directly overhead at a place is called the zenith point, usually denoted by Z . The angle between the zenith point and the position of the sun is called the zenith angle, generally denoted by θ. It is measured along the great circle passing through the sun, denoted by S and the zenith point, usually denoted by Z. The angle between the great circle containing the sun and the celestial poles on the one hand and the great circle passing through the poles and the zenith point, on the other hand is called the hour angle of the sun. It is usually denoted by H .

The celestial North Pole, denoted by P , the zenith point Z and the position of the sun, denoted by S form the corners of a spherical triangle. This is shown in the figure 1.

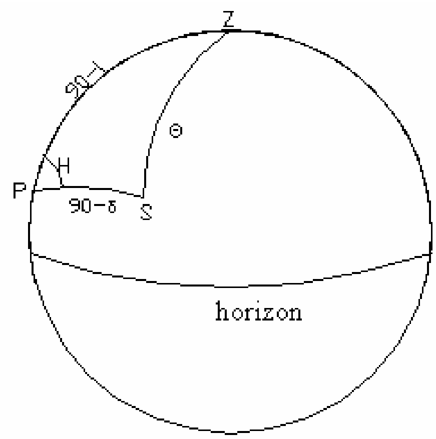


Figure 1. Position of the sun, the zenith, the North Pole on celestial sphere.

From spherical trigonometry (Roth, 1975) we have the identity:

$$Cos\theta = Sin\delta\ Sin l + (Cos\delta\, Cos l) Cos H$$

Where l is the angle of latitude of the place of observation.

By measuring the zenith angle θ as a function of the hour angle H, it is possible to extract $Sin\delta\ Sin l$ and $Cos\delta\, Cos l$ by a simple graphical method. Then, knowing the place latitude l, it is possible to determine the solar declination on the day of observation.

Experimental Procedure

A vertical rod attached to a flat steel base, a clock, a meter scale, a drawing sheet of paper and a pencil are the only things required for this experiment. The vertical rod is placed at a suitable place on the drawing sheet, which is kept on a flat horizontal surface. Observations are commenced when the sun is nearly two hours from local noon position.

The tip of the shadow formed by the vertical rod is marked on the sheet of paper (refer figure 2). The corresponding time is noted on the clock. The positions of the rod and the sheet should not change during the entire course of the experiment. After every 15 to 20 minutes of time, the position of the shadow of the tip of the vertical rod is marked on the sheet of paper. This process is continued till the sun is nearly two hours away from the local noon. In other words, some 12 to 16 readings are taken over a course of three to four hours. The length of the vertical rod from the horizontal plane is measured and recorded.

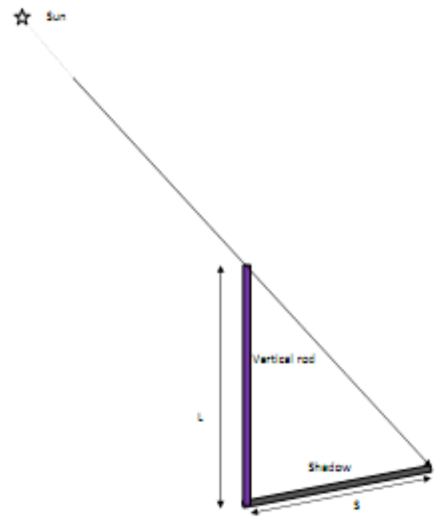


Figure 2: Schematic of shadow formed by a vertical rod

Tabulation and calculations

If the shadow of the rod is *S*, while its length is *L*, from geometrical considerations we can find the zenith angle using the formula:

$$\theta = \tan^{-1}\left(\frac{S}{L}\right)$$

A plot of shadow length is made as a function of time. The shadow length should be minimum at local noon, that is, when the hour angle is zero. Either graphically or by interpolation, the time corresponding to zero hour angle is determined. Let this be denoted by t_o. This can also be calculated by knowing local sunrise and sunset time. Then the hour angle for any other time t can be calculated from

$$H\,(\text{in degrees}) = \frac{\{(t - t_0)\text{ in minutes}\}}{60} \times 15^0$$

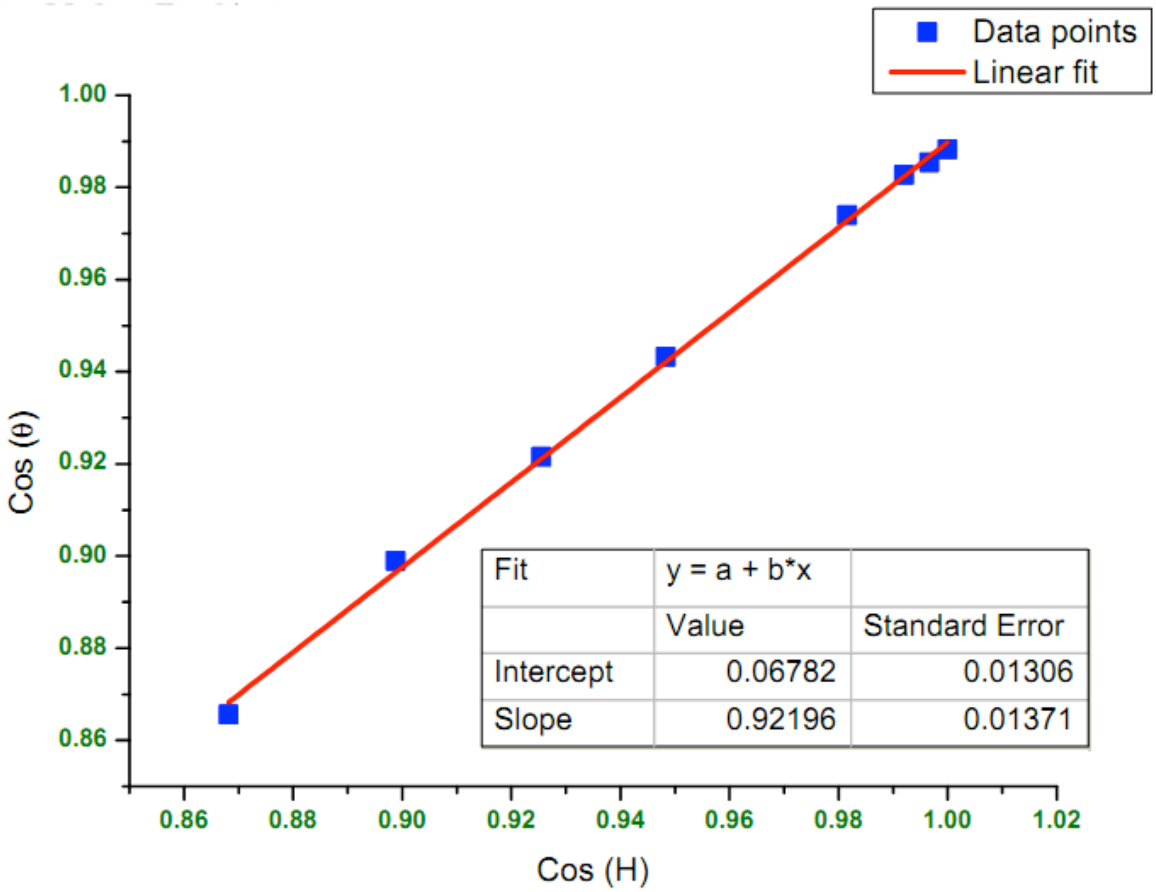
This follows from the fact that the hour angle of the sun changes by 15 degrees in one hour.

A plot of $Cos\,\theta$ is made against $Cos\,H$ with the help of the data points. A best fitting straight line is drawn through the points. The slope of the straight line gives $Cos\,\delta\,Cos\,l$, while the intercept on the $Cos\,\theta$ axis gives $Sin\delta\ Sin\,l$. Since the latitude of the place of observation is known, by inserting the value of $Cos\ l$ or $Sin\,l$, it is possible to determine the declination angle δ of the sun. It may be noted that both the place latitude l as well as the solar declination δ can be determined by knowing the slope and the intercept by solving slightly more complicated algebraic equations. The measured hour angles of the sun may be compared with those that are available in astronomical almanacs. Alternatively, if the rise time and setting times of the sun on that particular day are ascertained, it is possible to calculate the time for local noon and hence, the hour angles of the sun can be calculated from time measurements for the observed shadow readings. This will be useful whenever the shadow readings are not obtained for sufficient number of times.

Observations made on 26 July 2011, (Tuesday) at Mysore, Karnataka, India.

Length of the vertical rod used (L) = 159 cm
Sun rise time (from newspaper (Prajavani, 2011; PAC, 2011) = $6^h 9^m$
Sun set time (from newspaper) = 1
$8^h 52^m$
Hence, local noon time on the day = $12^h 30^m 30^s$

Time t	Shadow length S (cm)	Zenith Angle θ (degrees)	Time away from noon (min)	Hour angle (degrees)	$Cos\ \theta$	$Cos\ H$
$12^h 15^m$	26.5	9.46	16	4.0	0.9864	0.9976
$12^h 31^m$	24.5	8.76	0	0.0	0.9883	1.0000
$12^h 50^m$	27.5	9.81	19	4.75	0.9854	0.9966
$13^h 00^m$	30.0	10.68	29	7.25	0.9827	0.9920
$13^h 15^m$	37.0	13.10	44	11.0	0.9740	0.9816
$13^h 45^m$	56	19.40	74	18.5	0.9432	0.9483
$14^h 00^m$	67	22.85	89	22.25	0.9215	0.9255
$14^h 15^m$	77.5	25.98	104	26.0	0.8989	0.8988
$14^h 30^m$	92	30.05	119	29.75	0.8656	0.8682



Graph 1. Actual linear fit plot of Cosθ versus CosH

Calculations

Slope of the straight line from the graph = 0.92196

$$Cos\delta\ Cos\,l = 0.92196$$

$$Cos\delta = \frac{0.92196}{Cos(12^0 18^{'} 30^{"})} = 0.94366$$

$$\delta = Cos^{-1}(0.94366) = 19.32^0$$

Intercept of the straight line from the graph = 0.06782

$$Sin\delta\ Sin\,l = 0.06782$$

$$Sin\delta = \frac{0.06782}{Sin\ (12^0 18^{'} 30^{"})} = 0.3182$$

$$\therefore\quad \delta = Sin^{-1}(0.3182) = 18.55^0$$

The slight difference in the value of δ is may be because of error associated with vertical alignment of the rod and due to diffraction effects of sunlight.

The declination, in degrees, for any given day may be calculated from the approximate equation of Cooper (Rai, 2009) given by

$$\delta = 23.45\,Sin\left[360\times\frac{284+n}{365}\right]$$

Where n represents n^{th} day of the year. Thus July 26, 2011, is the 207th (31+28+31+30+31+30+26) day of 2011.
Hence $n = 207$.

$$\delta = 23.45\,Sin\left[360\times\frac{284+207}{365}\right] = 19.378^{0}$$

The value of declination obtained by our experiment also agrees well with the standard value given in in almanacs (TAMU, 2011), which is about 19.38^{0}.

Results and Discussions

The declination of the sun on 26 July 2011 is found to be nearly 19.32^{0}; this value is in good agreement with the theoretically estimated value from Cooper's formula as well as with the standard value given in in almanacs. The observational experiment discussed in the paper can be profitably performed by undergraduate students in physics/ astronomy / mathematics. The experiment is meant to teach students about the celestial sphere, astronomical coordinates systems, and the apparent motion of the sun during the course of year, actual noontime and its difference from the mean noontime used by clocks etc.

This is also a daytime astronomy experiment that can be performed within three hours and hence it can be performed during school / college hours under the supervision of instructors. Hence, it can be employed in courses where examination is required.

Most importantly, it can be performed with extremely low cost equipment. Hence, most ' third world ' countries can adapt it in their teaching curricula. The results of the experiment performed on a good sunny day agree very well with the formulae.

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