Equilibrium States and Transition Processes in Electric Circuits: Requirements for a deeper understanding

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ABSTRACT

Ohm's law and Kirchhoff's rules refer to stationary states and do not provide any indications of the always-present transition processes that connect these states and cause their respective setting. Through the use of suitable simulation programs these transition processes are accessible to classroom activities and allow a deeper and more coherent understanding of the ongoing processes.

Keywords: Electric circuit, stationary states, transition processes, simulation program for circuits, compartment model, simulation program for a transport line, transport equations.

STATIONARY STATES WITHOUT TRANSITIONS (a didactical problem)

The subject of "electrical circuits" is part of physics lessons in practically all countries. One finds a similar procedure in which only stationary states and the associated laws are dealt with, while the always present transition processes are ignored. An internet search under the keyword "transition processes in physics" does not provide any suitable results, except for a publication from the field of "Advanced Circuit Analysis" where transition processes are solved on the basis of differential and difference equations. (Student-Circuit 2015).

In a test carried out in different European countries students were asked the following question:

In a test carried out in several European countries high school students and university students were given the following task:

Given is an electric circuit that consists of a battery and a resistor in which a current is flowing, the strength of which is given by Ohm's law (figure 1).

***Figure 1**. Battery and Variable Resistor as Elements of an Electrical Circuit*

If the resistance is suddenly increased the current decreases in such a way that a new steady state is reached which in turn is given by Ohm's law.

Question: How does this process of transition from one stationary state to another takes place?

The result was clearly negative (Härtel 2001). The vast majority of our students had never been introduced to the existence of transition processes, nor had they heard of their existence. Transition processes in electrical circuits are usually very short and can only be determined experimentally with some effort. This fact may have been a reason to exclude these processes from consideration in classrooms.

If the goal of teaching physics is to foster on accurate observation and precise thinking, this traditional approach becomes problematic. It surely is not satisfactory if students are content with steady states and do not ask questions such as:

- How does the battery "know" that a resistor has been changed some distance away and that the current has to be adjusted?

or

- How fast does the current increase if a short circuit occurs somewhere in the circuit?

When only teaching steady states there is a risk that students learn that fast-moving processes are not important? Or they may learn that one can assume action at a distance in order to answer such questions

If students do not ask such questions, they should be made aware of the fact that there are gaps in the causal explanations of the electric circuit that can only be closed when transition processes are included.

Modern simulation programs offer a possibility to eliminate or at least to reduce these deficits, as shown below.

INSTRUCTIONS FOR CLASSROOM ACTIVITIES

Experimental Demonstrations

The existence of such transition processes can be demonstrated experimentally, at least for the special case of a circuit with a capacitor.

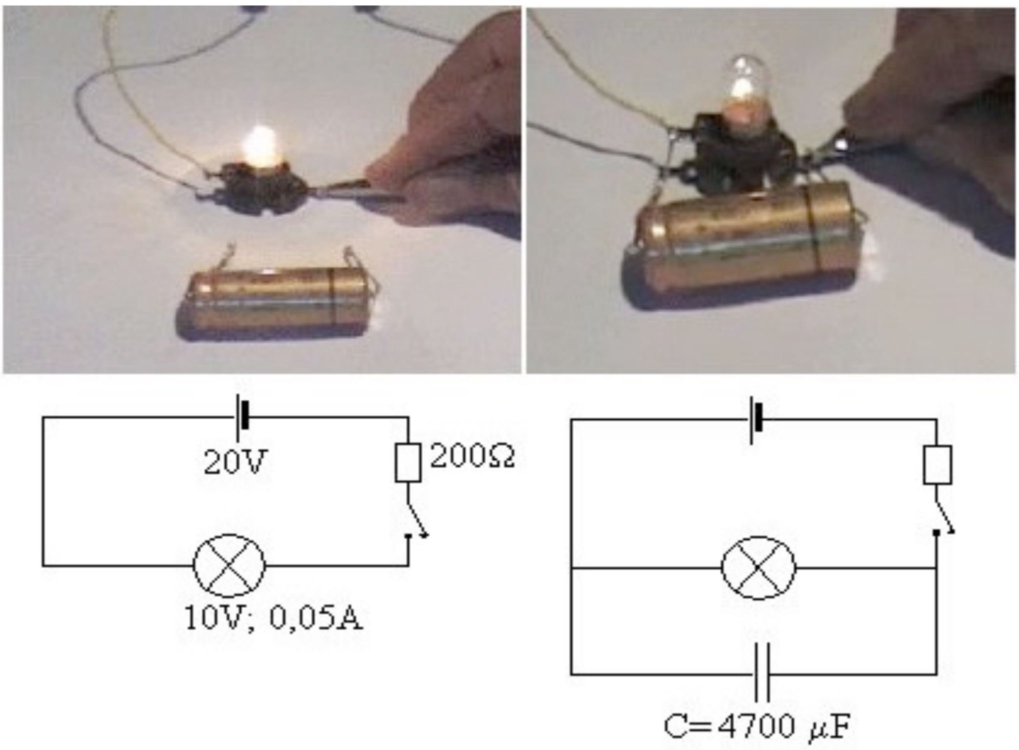


Figure 2. *Demonstration of a Transition Process. A light Bulb is Connected to a Voltage Source directly or with a Capacitor Connected in Parallel.*

If the light bulb is not connected directly to the voltage source (figure 1 left), but in parallel with a capacitor (right), the delay caused by the charging and discharging of the capacitor is clearly visible.

A video of this experiment can be found at: http://www.astrophysik.uni-kiel.de/~hhaertel/PUB/lamp-capacitor.flv

In order for this experiment to be convincing, the principle of how a capacitor works must be known. This in turn requires the knowledge of the existence of surface charges and the knowledge that in each electric circuit every individual conductor element not only conducts the current, but in addition represents a (usually rather small) capacitor due to its surface area. This has to be charged or recharged during each transition process.

In principle, the same process takes place in both experiments (bulb with and without capacitor). Even without the connected capacitor, there is a delay - albeit a very short one - since the surfaces of the conductor elements connecting the source and bulb have to be charged or discharged when the voltage source is switched on and off. Due to the capacitor connected in parallel in the above experiment, the surface to be charged or discharged is enlarged in order to make the transition process directly visible.

The experiment on the left in figure 2 therefore mimics the experiment on the right. The conducting elements that connect the source and the lamp have to be charged and de-charged whenever the lamp is switched on and off and this will cause a delay, albeit a rather brief one. The capacitor connected in parallel just increases the sur face of the conducting elements to make the transition process apparent.

Use of Simulation Programs in Physics Lessons

A quick search on the net under the keyword “simulations for physics lessons” confirms that there are a barely countable number of simulations that can be used in physics lessons. One example is the collection of PHET simulations, which stand out in their perfection and graphic design (PHET). At the same time, however, should be warned against too much optimism regarding the willingness of teachers to use such simulations in their lessons. Compared to the great importance of the real experiment many teachers find the use of simulations questionable. Another hurdle is the desire of teachers and lecturers to only use their own programs. Much additional material is required, such as a well-founded concept, a student-friendly operating instruction, reports on teaching experiences etc. Without such additional materials, you risk “to re-invent a flat tire” (Redish, 1999).

The simulation program CLOC (Conceptual Learning of Circuits) presented below is aimed primarily at teachers who have recognized the importance of transition processes for a deeper understanding of the electrical circuit or who agree to this meaning. The program shows in a qualitative and coarse form a delay in the display of a new state of equilibrium. Further materials such as videos explaining the operation of the user interface, documentation and sample tasks are available (Härtel, 2001).

The program CLOC - Conceptual Learning of Circuits

The program offers the user the possibility to create his or her own circuits from the elements voltage source, resistor, switch (interactive) and line element. The current strength is displayed graphically and numerically. The distribution of the potential along the entire circuit is shown in a three-dimensional representation in a separate window (Härtel, 2002).

Tony Minich (2005) also pursued the idea of using the third dimension to visualize the course of the electrical potential. Minich had the students simulate the course of the potential with the help of real components. Such practical activities may be more impressive for students than computer simulations, but they are time consuming and hardly adaptable. Balta (2015) also uses a 3D representation to visualize the course of the potential difference along a circuit and reports on positive learning successes.

The program CLOC can be run as a standalone Java application.

The special type of algorithm implemented makes this program interesting in respect to transition processes. This algorithm is based on the so-called container model. In this model, resistors are treated as such, while the neighboring conductor elements are modelled as containers. Similar to capacitors connected in parallel this modelling causes that transition processes are slowed down. (A brief description of this model can be found in the appendix).

The following figure 3 shows the user interface of CLOC where an interactively created circuit is shown. In the window at the top left, the circuit is shown in a top view together with both a graphical and a numerical display of the current. The upper right window shows the same circuit with an interactively closed switch. The three figures below show the potential distribution along the same circuit in a 3D representation. This window can be rotated in the horizontal plane.

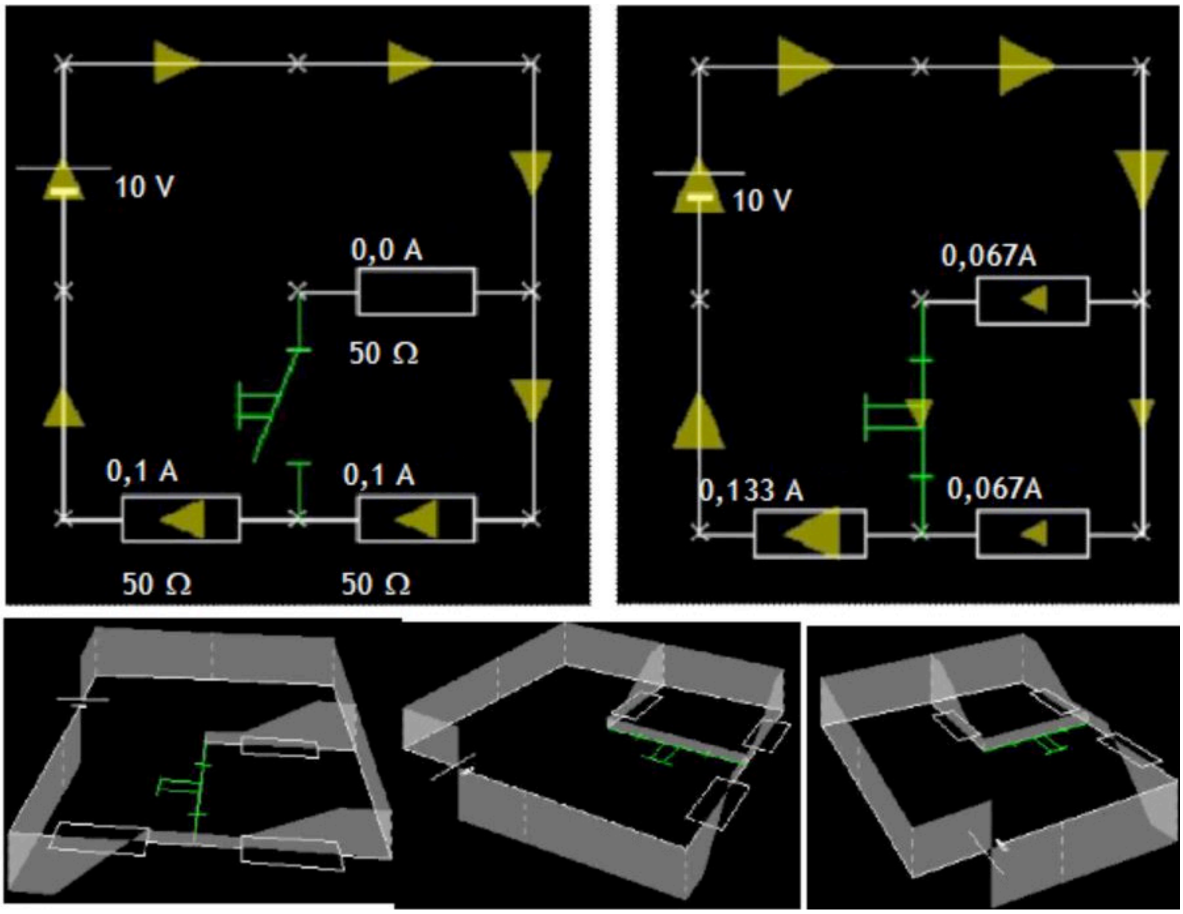


Figure 3. *Screen Displays of the Simulation Program CLOC (See text). The Separate Window Showing the Circuit in 3D Can Be Rotated continuously.*

Criticism of the container model can be raised in two ways.

- On the one hand, the delay shown corresponds only approximately to the conditions in a circuit in which capacitances dominate. This means that reflection processes are not recorded in the container model, which are particularly dominant in circuits with the usual small line capacities.
- Secondly, each connected conductor element is modelled as a single container, which means that there are no propagation processes along such conductor elements.

These restrictions appear to be acceptable if the transition processes made visible with the CLOC program are viewed as an approximation and primarily as a didactic tool in order to highlight the transition processes that are otherwise always bypassed.

A second simulation program, called "TL Transport Line" (presented below), does not suffer under these restrictions. The implemented algorithm allows to precisely calculate all transition processes for any specific situation with given initial and boundary conditions. Thus, the highly exaggerated and coarse transition processes “à la CLOC” can be seen as an intermediate step to demonstrate realistic situations

The Simulation Program "TL Transport Line"

The program "TL transport line" (figure 4) provides a visualization of the transition processes on a twin line (Härtel, 2010).

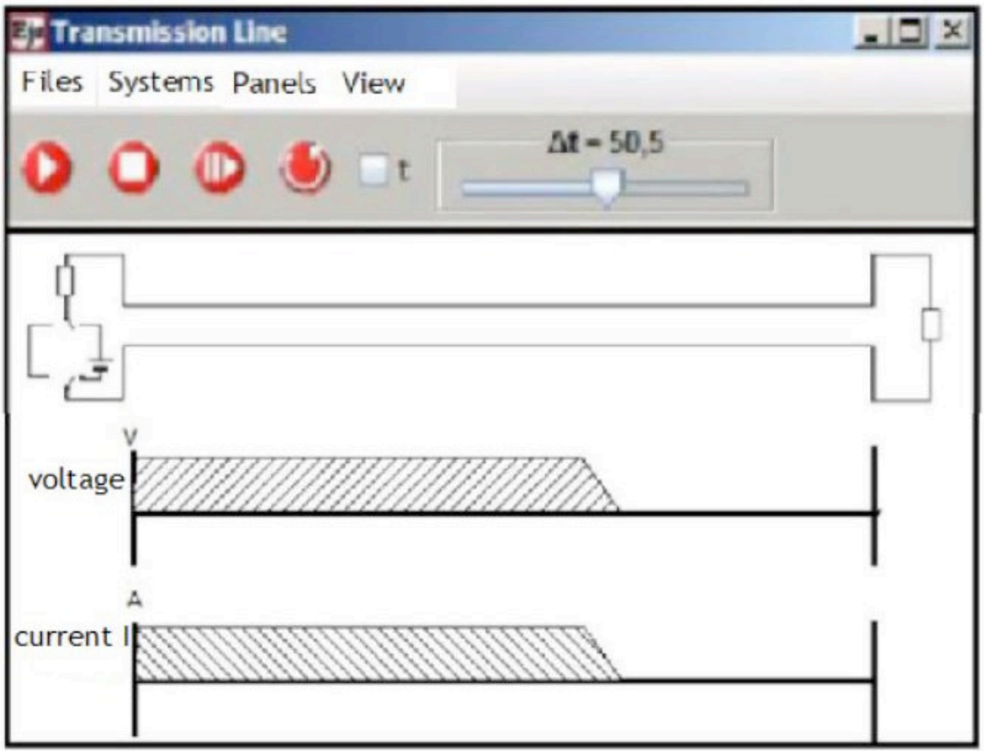


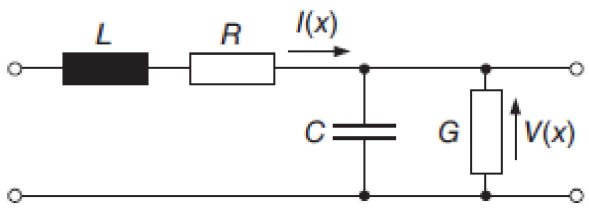
Figure 4. *Transition Process After Connecting a Voltage Source to a Twin Line*

The algorithm used provides the numerical solution of the following two differential equations, also known as "transport equations".

$$(1) \qquad \frac{\partial}{\partial x} I(x,t) = -C\frac{\partial}{\partial t} V(x,t) - V(x,t)G$$

$$(2) \qquad \frac{\partial}{\partial x} V(x,t) = -L\frac{\partial}{\partial t} I(x,t) - I(x,t)R$$

These equations are obtained by modelling a twin line as a series of identical elements of the following form (figure 5):



L: Inductivity of the element
R: Loss resistance of the 2 conductive elements
C: Capacity between the conductors
G: Leaking resistance (G = 1/R)

Figure 5. *Model of an Element of a Twin Line*

In case of the TL simulation program, the modelled twine line consists of 200 such elements. The equations above are derived by applying the knot rule (equation 1) and the mash rule (equation 2) to each single element (with L, R, C, G per unit length). The same result is derived by solving Maxwell´s equations in one dimension.

The functions V (x) and I (x) result from the numerical solution of these differential equations for all 200 elements of the line, together with the corresponding boundary and initial conditions. Their change over time is shown in the simulation program TL in corresponding diagrams parallel to the double line (see figure 4).

The assumption for this one-dimensional solution is a long linear line where changes in the vertical direction can be neglected. The implemented algorithm has proven to be stable for all boundary and initial conditions without showing any unrealistic effects due to improper numerical methods. The algorithm was originally developed by Zvonko Fazarinc. The program was written in Java by Francisco Esquembre and can be used as a Java standalone application.

The program TL was originally developed to support teaching of transition processes in linear systems. In relation to series and parallel circuits, of interest here, this program gains its importance from the possibility of representing these two circuit types where all transition processes can be visualized in a realistic manner based on the one-dimensional solution of Maxwell´s equations (figures 6 and 7).

The program TL offers the possibility to activate either a serial or a parallel circuit and to display all possible transition processes in real time.

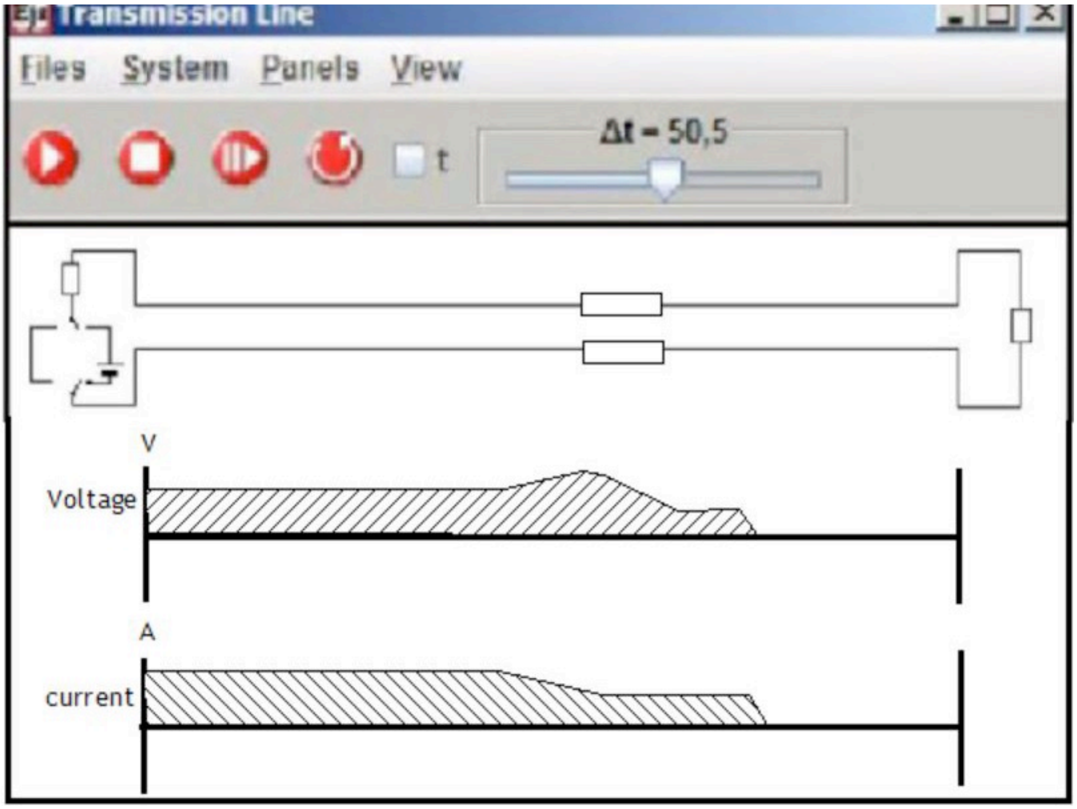


Figure 6. *Transition Process for a Serial Circuit (Resistors on Both Lines Plus Load)*

The two symmetrically arranged resistors in the middle of the twin line together with the load resistor at the end form a serial circuit with variable resistors. Reflection processes will occur upon impact of any current/voltage slope which can be traced until a final stationary state is reached.

Additionally as seen in figure 7 the resistance in the middle of the twin line can be replaced by a conductive area between the two lines of limited width and variable resistance. Together with the load at the end of the line, this configuration represents a mixed circuit, in which all ongoing transition processes can be monitored until steady states are reached.

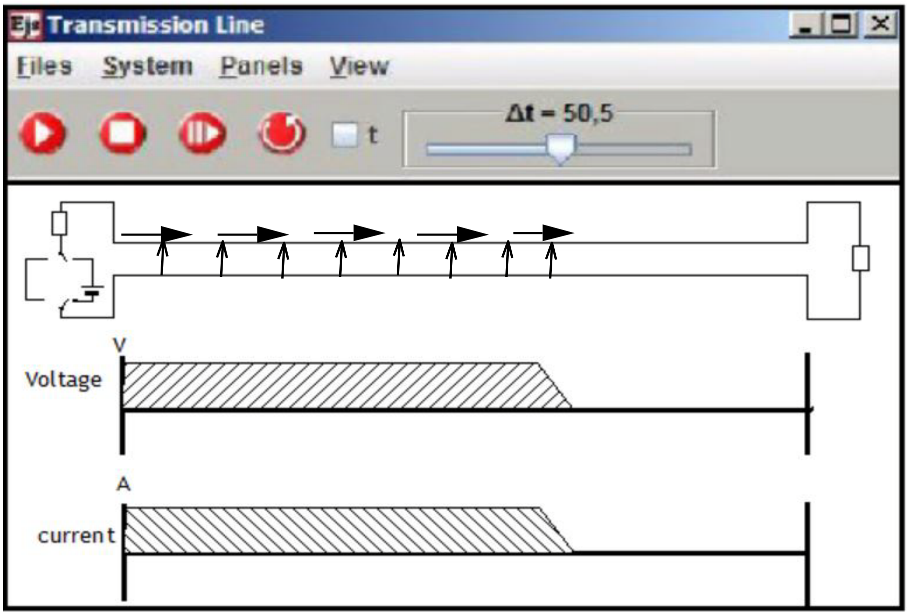


Figure 7. *Transition Processes along a Parallel Circuit (Conductive Region between both Conductors Plus Load)*

A further visual support for understanding and concept development is given by a display of voltage and current conditions along the line in form of arrows (figure 8).

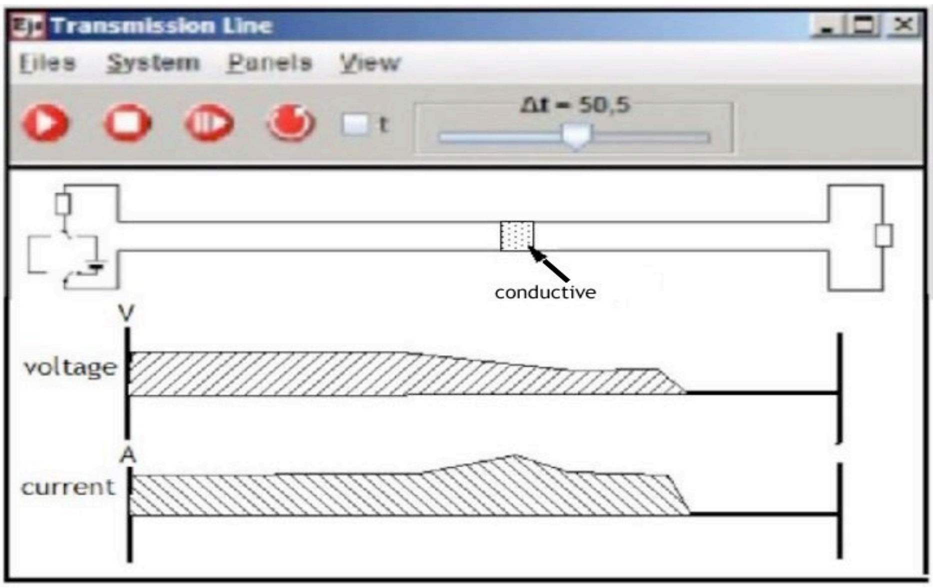


Figure 8. *Indication of Direction and Strength of the Electric Field (Open Arrow) and Intensity of Current (Full Arrow) along a Twin Line.*

CONCLUSION

The electrical circuit in practical use is a rather simple system. In general, it is sufficient to know whether or not a voltage is applied and whether a current is flowing. To compute stationary states does not require particular mathematical skills.

However, the task of explaining and understanding how the electric circuits actually functions is challenging.

Energy is transferred via force and motion and not via energy enriched matter. The consequences of this energy transfer and the basic relations of the system must be understood.

Students must recognize why surface charges build up on each current carrying conductor and why these surface charges in interaction with the conduction electrons are responsible for the flow of a constant current and the possibility that a steady state can be achieved.

Finally, students must understand that stationary states cannot exist without transition processes, whose characteristics should in principle be known.

Which of these aspects should be selected for classroom activities and in what detail is left to the teachers, whenever possible supported by research.

Annex

The CLOC-algorithm is based on the so-called container model. A resistor is treated as such, while the adjacent circuit elements are modelled as containers (figure 9).

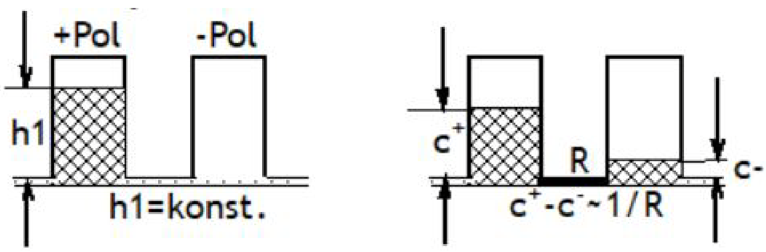


Figure 9. *Model for a Battery (Left) and Resistor R with Two Connections (Right)*

After decomposing a circuit into single elements, the program determines continuously the difference in content of all adjacent containers (c^+-c^-). A certain percentage of this difference (proportional to the connecting resistor R) is moved during each calculation step from one container to the next. Finally a steady state is reached for all containers along the circuit. An exception is how the battery is modelled. It maintains a predetermined difference in the contents of its adjacent containers (fig.9 left).

After starting the simulation, the difference in content gradually spreads to all connected containers. Depending on the number of components and the speed of the computer a transition process is visible for a short period of time, until the system reaches a final stationary state.

This final state is in every case consistent with experimental measurements. The stationary final state achieved with the container model could be calculated directly with the help of Kirchhoff´s rules and Ohm´s law. This method is used in practically all known simulation programs. From a teaching perspective this is not ideal however, because it relates only to steady states that seem to be reached instantaneously whenever a system change occurs.

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