Dual Purpose Measurements with Displacement Sensors

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Abstract

In this paper, we show a laboratory experience describing the possibility to build a sensor using a coil to measure small thicknesses of materials with the possibility of measuring temperature simultaneously, with the same built sensor. Its operation is based on the following facts: An electric current (a.c), flows through a coil and a magnetic field appears producing self-induction characterized by an electromotive force induced in the coil when a conductive piece is situated in front of the coil. That permits us to obtain information about the distance between the coil and the conductive piece. When this separation thickness is changing, the magnetic field around the coil changes, because the self-induction coefficient (L) is also changing. Using resistance and impedance measurements (voltage in our case) in the coil, an expression has been obtained for the determination of the thickness of a non-conductive sheet placed between a metallic plate and the coil. Calibration measurements of resistance with temperature have been obtained. The thermodynamic analysis is also presented showing the equation of state of the system between the voltage, temperature, and the thickness of the non-conductive sample. The linear thermal expansion coefficient of the sample is also determined.

Keywords: Laboratory experience, dual-purpose measurement, displacement sensor.

INTRODUCTION

The main objective of the paper is to assess the possibility of measuring two different magnitudes of a flat sheet. Namely, their thickness and temperature, using one simple and cheap self-built sensor. This sensor is important in some applications to know the temperature to accurately measure the thickness of the polymeric membrane, where the knowledge of its thickness is crucial. Particularly, in the determination of the permeability and diffusion coefficient measurements of membranes. For the fabrication of the sensor, a coil is placed in front of a conductive material, and an alternating electric current (a.c.) flows through it (Serway, 1996; Kosov, 1964). The distance between the coil and the conductive material can be measured employing the electromotive force induced in the coil, ε_{ind}, according to Faraday-Lenz law (Smith, Wiedenbeck, 1959). Some commercial devices used for the accurate measurements of the thickness of thin sheets utilize systems similar to those described here. Nevertheless, the problem appears when the environmental temperature changes, and so the resistance of the coil because that produces a modification in voltage lectures and the thicknesses measured are wrong. Due to this, it is convenient to study the relation between these magnitudes, thickness, emf, resistance, and temperature.

With this aim, we are proposed a lab experiment that allows students to understand some important electromagnetic properties is described. The experiment consists in measuring the resistance (R) of a small cylindrical coil as a function of temperature (T) (Keszei, 2012). By taking into account that the resistance of the coil does not depend on its position concerning other conductive pieces, the calibration curve relating resistance with temperature is obtained. Another phenomenon considered here is that the impedance of a coil is affected by the proximity of a conductive mass due to changes in the self-inducted current (Reitz, Milford, Christy, 1979). The impedance of the coil is measured as a function of its distance (Th) to a metal plate. As a result of the two phenomena, the calibration of Th versus Z and T is accomplished. Therefore, the experiment allows the accurate determination of the temperature and thickness of thin non-conductive material sheets from resistance and impedance measurements. Once, the different thicknesses and the respective temperatures have been established, the coefficient of linear thermal-expansion can be obtained, as well as the relationship between the two different variables of the system with the temperature, i.e. the equation of state.

EXPERIMENTAL PROCEDURE

Materials

Cooper coil: A cylindrical coil of 4 mm diameter and 3 mm length was prepared by winding 150 turns of copper wire of 0.1 mm of diameter around an appropriate Teflon cylinder of 0.8 mm diameter. The structure of the coil was fixed with glue.

Driver Weir Pumps LTD, Electronic Division, mod. DDNA 2 (L): This device is used to measure impedance though this quantity can be measured with any other system, for example, an impedance-meter. The driver measures the impedance produced in the coil by an a.c. current. The output of the driver is a d.c. a voltage proportional to the impedance. The driver is fed with an 18 V d.c. power-source. *18 V DC power source for the driver*. *Voltmeter*: The voltage (proportional to the impedance) is measured with a voltmeter with a precision is 0.01 V. A multimeter Hewlett Packard 34401A with a precision of 0.0001 V was used for the described experiment. *Ohmmeter*: An ohmmeter with a precision of at least 0.01 Ω is needed. In this case, a Hewlett Packard 34401A multimeter with a precision of 0.001 Ω was used. *Oven*: The experimental device integrated by the coil, metal plate, non-conductive sheets, and cables is maintained at a controlled temperature inside an oven. An oven with a precision of 1 ºC is recommended. *Paper or polymeric membrane separation*: The coil is maintained at an exact distance of the metal plate with normal paper sheets whose thickness is further exactly calibrated. It should be pointed out that any kind of non-conductive flat sheet can be used if their thickness. *Metal plate:* A 10 x 10 x 1.5 cm iron plate can be used as conductive mass.

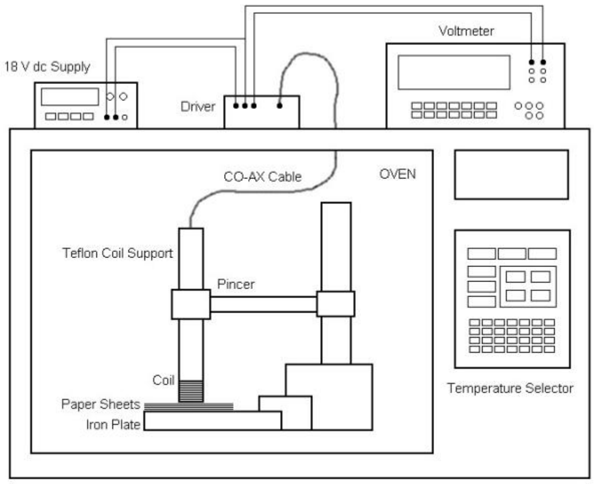

Figure 1. *Experimental Setup*

Measuring System and Process

Determination of The Resistance

The use of the coil as a thermometer for measuring the temperature of a piece requires the knowledge of the thickness of such a piece. On the other hand, the temperature of the coil also allows the determination of the thickness of the piece since impedance depends on it. The use of the resistance of the coil to determine its temperature requires its calibration first. In principle, it is very difficult to measure the resistance of the coil with an accuracy better than 0.01 ºC due to the resistances of both the cables, about 0.02 ºC in our case, and the contact points about 0.3 ºC in our experiments. Fortunately, it is a more important good reproducibility than accuracy. Therefore, by performing the measurements on the pieces in the same way as the calibrations good values of temperature can be obtained. It is recommended the use of tin solders whenever possible or the utilization of the same type of connectors if solders are not available. Some calibration resistance measurements carried out in our laboratories are shown in table 2. Note that the experimental values were obtained with the coil near to the metal.

Measuring Voltage

Now the experiment is ready to calculate the temperature of the coil and to make calibration measurements of impedance for different distances and different temperatures. The coil is kept over the metal plate separated from it by a stack of paper sheets. By changing the number of sheets, the distance between the coil and the metal plate is varied. The metal plate, coil, and sheets are placed inside an oven and we can proceed with the measurement of the impedance of the coil at different temperatures. In our case, we have measured the output voltage of our driver instead of impedances (remember that, in our device, voltage is proportional to impedance).

Theory

Since the current intensity in the coil varies with time (t) according to the equation:

$$I = I_0 \cdot \sin(\omega \cdot t) \qquad (1)$$

where I_0 is the amplitude of the alternating current and ω is its angular frequency. The magnetic field produced by the coil is also time-dependent (Jefimenko, 1989). The alternating field induces an electromotive force in the coil, ε_{ind}, that, according to Faraday-Lenz law,

$$\varepsilon_{Ind} = -N\frac{d\int B \cdot dS}{dt} = -L\frac{dI_{Ind}}{dt} \qquad (2)$$

where I_{ind}, denote the current intensity induced into the coil. This phenomenon is called self-induction because ε_{ind} arises from the variation of magnetic flux in the circuit and therefore, ε_{ind} is called the self-induced electromotive force. On the other hand, Ohm's law can be written as

$$\varepsilon_{Ind} = I_{Ind} \cdot Z \qquad (3)$$

where Z is the coil impedance and ε_{ind} and I_{ind} are, respectively, the effective values of the self-induced electromotive force and the self-induced current. The impedance is related to the self-inductance L through the following expression

$$Z = \sqrt{R^2 + (L\omega)^2} \qquad (4)$$

where R is the ohmic resistance of the coil.

Our system is useful by the fact that changes in the distance from the coil to the metal plate changes the impedance of the through. These changes are due to variations in the magnetic field that changes the self-induction coefficient (L).

Calculate analytically the exact L is very difficult due to the complexity of the integrals involved in the process. In the next paragraphs we have made a calculus of this L but making important simplifications that permit us to obtain a value for L not very far away from the real one. Let us consider a cylindrical coil of N (150 in ours) spires. The infinitesimal number of spires in an element of area $dS = drdx$ is given by:

$$\frac{N}{(b-a)\cdot h} dr \cdot dx \tag{5}$$

where the axis x is taken normal to the cylinder. Taking into account that each spire transports an electric current I, the spires contained into the element of the area considered should transport the current:

$$dI = \frac{N \cdot I}{(b-a)\cdot h} dr \cdot dx \tag{6}$$

According to Biot y Savart law the module of a magnetic field produced using current dI in an arbitrary point of the coil axis (axis x), is given by:

$$B_{x_0} = \frac{\mu}{2} \frac{N \cdot I}{(b-a)\cdot h} \int dx \int \frac{r^2 dr}{(r^2+x^2)^{\frac{3}{2}}} \tag{7}$$

Integration of this equation for the radial component leads to:

$$B_{x_0} = \frac{\mu}{2} \frac{N \cdot I}{(b-a)\cdot h} \int_{x_0-\frac{h}{2}}^{x_0+\frac{h}{2}} \left[\ln \frac{\left[\sqrt{b^2+x^2}+b\right]}{\left[\sqrt{a^2+x^2}+a\right]} - \frac{b}{\sqrt{b^2+x^2}} + \frac{a}{\sqrt{a^2+x^2}} \right] \mathrm{dx} \tag{8}$$

Where x_0 has been set in the center of the coil and h is the coil length. Thus, integrating eq. (8):

$$B_{x_0} = \frac{\mu}{2} \frac{N \cdot I}{(b-a)\cdot h} \left[x \ln \frac{\sqrt{b^2+x^2}+b}{\sqrt{a^2+x^2}+a} \right]_{x_0-\frac{h}{2}}^{x_0+\frac{h}{2}} \tag{9}$$

The values calculated for the magnetic field utilizing eq. (9) and those obtained experimentally for a current intensity of 0.3 A are shown in table 1 and plotted in figure 2. In this figure you can see the calculated values for the magnetic field, B. These values have been compared with other measured with a tesla meter, and the results are good enough.

Table 1. *Values of the magnetic field measured (Bm), and calculated using Eq. (9) (Bc), for a current intensity of 0.3 A*

Xo (mm)	Bm (Gauss)	Bc (Gauss)
0		148
0.5		142
1		125
1.5		87
2		49
2.5	32	28
3	17	17
3.5		11
4	9.7	7.4
4.5		5.2
5		3.8
5.5		2.9
6	2.1	2.2
6.5		1.7
7		1.4
7.5		1.1
8	0.7	0.9
8.5		0.8
9		0.6
9.5		0.5
10	0.3	0.5

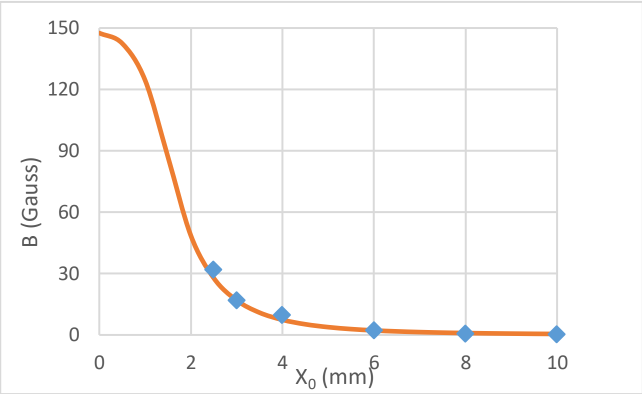

Figure 2. *Variation of the calculated magnetic field given in eq. (9) with the distance (red line). Blue dots represent the experimentally measured magnetic field*

Now we have to do the most drastic approximation, we will assume that B is constant everywhere inside the coil. This approximation is very good when you are far away from the limits of the coil. In these conditions, we can calculate the total magnetic flux through the coil as:

$$\Phi = B \cdot N \cdot \overline{S} \qquad (10)$$

where B is the average magnetic field inside the coil, N the number of spires, and $\overline{S}$ is the average area of the spires of the coil defined as:

$$\grave{S} = \frac{1}{(b-a)} \int_a^b \pi \cdot r^2 \cdot dr \qquad (11)$$

Knowing the flux, then the self-induction coefficient L can be calculated as:

$$L = \frac{\Phi}{I} \qquad (12)$$

Using the results of eq. (9) we can calculate an average magnetic field inside the coil. The value obtained was B = 129.3 Gauss. Taking into account the eq. (11) the average area calculated is $\overline{S}$ = 5.19·10-6 m2. Like I = 0.3 A, then we can obtain a calculated value of L:

$$L = \frac{129.3 \cdot 10^{-4} \cdot 150 \cdot 5.19 \cdot 10^{-6}}{0.3} = 33.7 \cdot 10^{-6} H \qquad (13)$$

To know the goodness of this result we have measured L directly and we have obtained a real value of L = 21.7 μH (in the interval of frequencies from ω = 1 KHz to ω = 1 MHz). Since Lω is small (with a working frequency ω = 1 KHz) compared with the resistance (3 to 4 Ω) then the variations of Z with temperature are nearly the same that variations of R with this parameter (see eq. (4)). On the other hand, since R doesn't depend on the distance from the coil to the metal plate, the variations of the impedance, Z, with this parameter are due to variations in the self-induction coefficient, L.

RESULTS AND DISCUSSION

The results of the calibration measurements of resistance are collected in table 2. Notice that the values are practically the same for measurements with or without the metal plate. This result is expected because the resistance does not depend on the external effect of the metal on both the magnetic field and the self-induction.

Table 2. *Measurements of resistance for different temperatures*

T (°C)	Coil-Air R (Ω)	Coil-Fe R (Ω)
25	3.363±0.004	3.365±0.004
50	3.681±0.004	3.686±0.004
75	4.006±0.004	4.010±0.004
100	4.334±0.004	4.330±0.004

The results also show that the resistance of the coil is a linear function of temperature. By using least square analysis, it is found that

$$T\ (^{\circ}C) = (77.66 \pm 0.13)\ R\ (\Omega) - (236.3 \pm 0.5) \qquad (14)$$

with a correlation coefficient better than 0.99999. This calibration can be used because in the most unfavorable situation when the metal plate is located far away from the coil, the maximum perpetrated error is less than 1%. To calibrate the thickness of the paper sheets, the thickness of different stacks has been plotted against the number of sheets of each stack in figure 3.

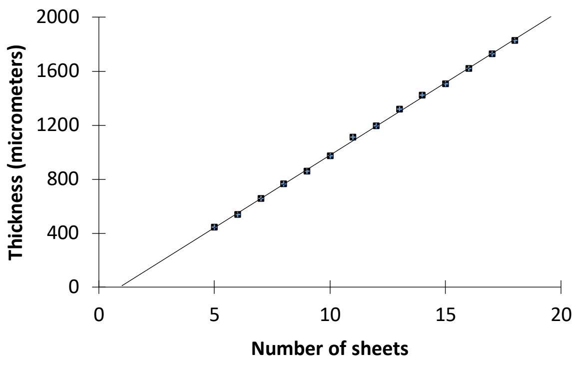

Figure 3. *The thickness of different stacks of paper sheets. The dots are the experimental values determined by the coil and the line represents the linear fit Th (μm) = 107.4 x - 97.1*

By fitting the results by the least square analysis the average thickness of each sheet was found to be 107.4±0.8 μm. The fact that the straight line does not intercept the origin of the coordinate axes at the origin is due to the error perpetrated in the determination of the thickness of the stacks caused by the flexibility of the sheets. However, this error is not important if the impedance is measured by disposing of the paper sheets in the same way as that used in the calibration measurements. Notice that commonly 10 to 18 paper sheets are used in the experiments, so the error involved in the determination of the thickness of each sheet is at least one-tenth of that corresponding at the origin. In figure 4 the voltage (remember proportional to impedance) at 25 ºC and 100 ºC, the two limits of temperature range measured, is plotted against the distance from the coil to the metal plate. It can be seen that there is a linear relationship for thickness lying in the range from 977 to 1836 μm. So that the coil can be used as a sensor in this interval. Out of this range, our driver is saturated. If we want to measure a thickness out of this range the coil must be changed for another with a different number of spires or to measure impedance by another method.

From both the linear relation voltage against thickness (Th) in the range, 977 to 1836 micrometers and the linear relation between impedance and temperature the following equation relating the voltage as a function of temperature and thickness is obtained

V (V) = 1.0348 + 0.0055 x T (ºC) + 0.0088 x Th(μm) - 1.0929 x 10^{-5} x T (ºC) x Th (μm) (15)

This equation is represented in Figure 4 as a black line for 25 ºC and 100ºC. Notice the good correlation between measured and calculated values in the linear range.

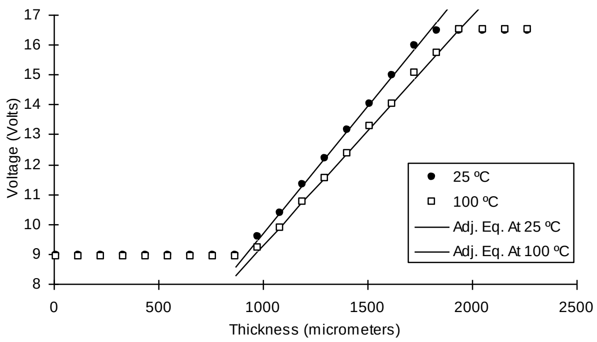

Figure 4. *Output voltage vs. distance between the coil and the conductive piece at 25 (●) and 100 ºC (□). The lines represent the linear fit for the range of thicknesses between 1000 to 2000 μm.*

Thermodynamic Analysis

By substituting equation (14) into equation (15), the thickness (Th), is obtained as a function of V and R. The pertinent expression, for the determination of the thickness of a non-conductive sheet (into our interest range) from resistance and impedance (voltage in our case) measurements are given by,

$$\mathrm{Th}(\mu m) = \frac{V(\mathrm{V}) - A - BT}{C - DT} \tag{16}$$

Where A = 1.0348 (V), B = 0.0055 (V/ºC), C = 0.0088 (V/μm) and D = 1.0929x10^{-5}(V/ºC/μm).

Now, considering the partial derivatives and its second partial derivatives respect V and T, respectively

$$\left(\frac{\partial T_h}{\partial V}\right)_T = \frac{1}{C-DT} \tag{17}$$

$$\frac{\partial^2 T_h}{\partial T \partial V} = \frac{D}{(C-DT)^2} \tag{18}$$

$$\left(\frac{\partial T_h}{\partial T}\right)_V = \frac{D(V-A-BT)}{(C-DT)^2} - \frac{B}{C-DT} \qquad (19)$$

$$\frac{\partial^2 T_h}{\partial V \partial T} = \frac{D}{(C-DT)^2} \qquad (20)$$

Then the following condition is fulfilled

$$\frac{\partial^2 T_h}{\partial T \partial V} = \frac{\partial^2 T_h}{\partial V \partial T}, \qquad (21)$$

Then the relation $T_h = T_h(TV)$ is an equation of state (Callen, 1985).

Evaluation of Thermal-Expansion Coefficient

Using equation (16), the thickness (Th) vs T is plotted. In this plot, nine curves are shown, each one with its respective voltage that remains constant. (See Figure 5). The value of the thermal expansion coefficient (α) is the slope of the line at a constant voltage value divided by the initial thickness with which the line starts. The average value representative of our sample is estimated by:

$$\alpha = 9.231x10^{-4}\frac{1}{°\mathrm{C}}$$

within an uncertainty of less than 10%

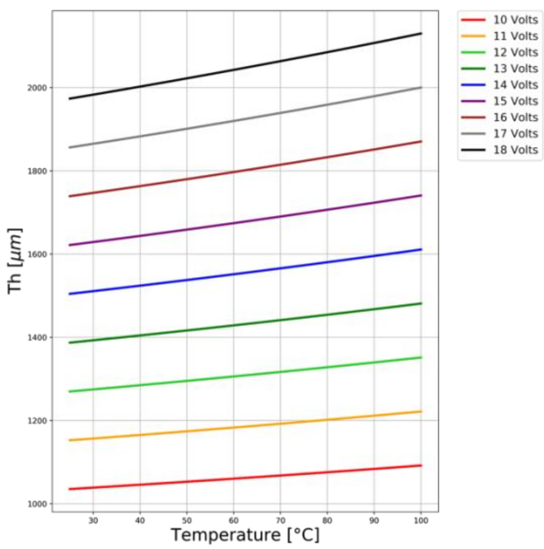

Figure 5. *Thickness vs. Temperature between 25 and 100 ºC. The lines represent the results of equations (16), for different voltages*

CONCLUSIONS

We show how to build a device very simple and cheap useful for measuring temperatures and small thicknesses, by measuring resistance and impedance. We also observed that the variations of the coil resistance with temperature are linear, and the impedance variations with the separation length to a metal plate are nearly lineal in the range analyzed in this study.

The values obtained by the theoretical expression for the magnetic field in the axe of the coil are in very good agreement with measured ones, as well as the self-induction coefficient of the coil. The data obtained of voltage and temperature measurements, in terms of the thickness of the sample, are fitted to linear expressions with a low percentage of error, (see equations 14 and 15). From these relations, the equation of the sample thickness concerning the voltage and temperature is obtained, and it is shown that this relationship is a thermodynamic equation of state. Then, the representative thermal-expansion coefficient of the sample is obtained.

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